

Acceptance Sampling (Variables)

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Summary

When lots containing a relatively large number of items require inspection, acceptance sampling plans can provide reasonable protection against shipping or receiving an unacceptable fraction of non-conforming items without inspecting 100% of the lot. The **Acceptance Sampling (Variables)** procedure generates acceptance sampling plans for situations when items can be measured and the measurements compared against established specification limits. In such plans, a sample of size n is drawn from a lot of N items and the lot is accepted if the sample mean lies no closer than k standard deviations to the nearer specification limit.

STATGRAPHICS generates three types of acceptance sampling plans:

- **OC Plans** - plans that control the alpha and beta risks, i.e., the probability of accepting a bad lot and the probability of rejecting a good lot. For such a plan, “good” and “bad” must be well-defined.
- **AOQL Plans** - plans that minimize the average outgoing quality limit, i.e., the maximum fraction of non-conforming items accepted on average. Such a plan requires 100% inspection and rectification of all rejected lots.
- **LTPD Plans** - plans that minimize total inspection while controlling the risk of rejecting a bad lot, where “bad” must again be well-defined. Such a plan also requires 100% inspection and rectification of all rejected lots.

Sample StatFolio: *acceptvariables.sgp*

Sample data:

None.

Acceptance Sampling Plans for Variables

In a variables plan, a sample of n items is taken from a lot of N , and each item is measured. The sample mean $\bar{x}$ and the sample standard deviation s are calculated. The lot is then accepted or rejected according to the following rules:

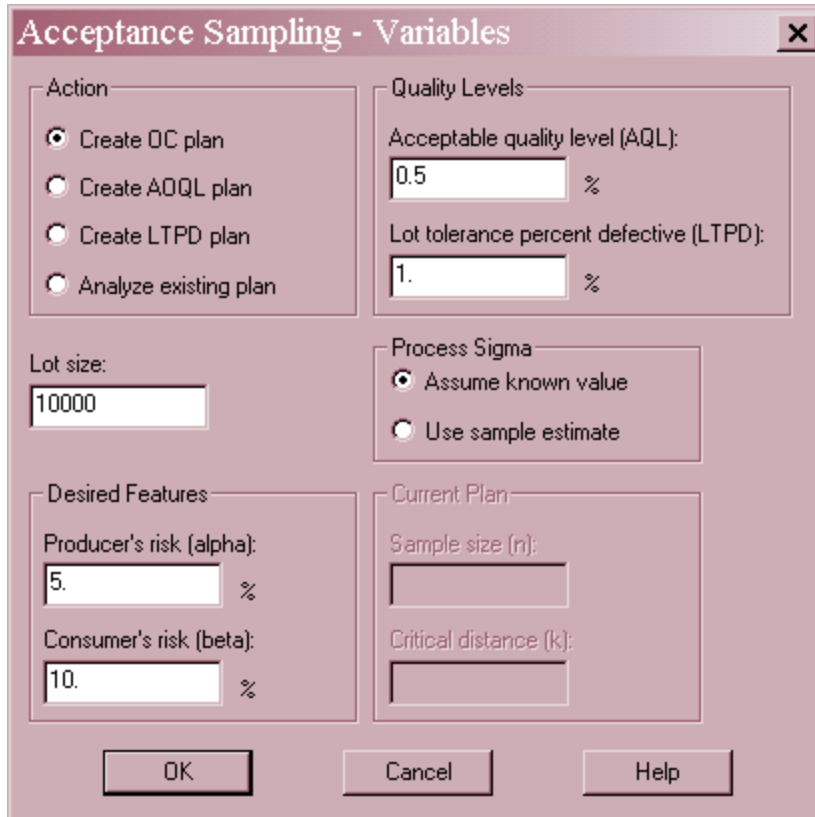
- If the sample mean $\bar{x}$ lies no closer than a critical distance $k\sigma$ from the nearer specification limit, the lot is accepted.
- Otherwise, one of two actions is taken:
 1. If the lot is *rectifiable*, then all remaining items in the lot are measured. Any non-conforming items are replaced by conforming ones to yield N conforming items.
 2. If the lot is not rectifiable, the lot is rejected without further inspection and returned to the producer.

In the discussion that follows, several terms are important:

1. AQL = *acceptable quality level*, defined as the poorest level of quality that the consumer finds acceptable *on average*.
2. LTPD = *lot tolerance percent defective*, defined as the poorest level of quality that the consumer is willing to tolerate in any given lot.
3. OC(θ) = *operating characteristic*, defined as the probability that a sampling plan will accept a lot when the lot contains a fraction θ of non-conforming items.
4. AOQL = *average outgoing quality limit*, defined as the maximum percent of defective items accepted by a given sampling plan assuming that all rejected lots are subjected to 100% inspection and all non-conforming items in such lots are replaced with conforming items.

Data Input

The data input dialog box defines the desired features of the sampling plan.



The dialog box titled "Acceptance Sampling - Variables" contains the following fields and controls:

- Action:** Four radio buttons: "Create OC plan" (selected), "Create AOQL plan", "Create LTPD plan", and "Analyze existing plan".
- Quality Levels:** Two text boxes: "Acceptable quality level (AQL):" with value "0.5" and "Lot tolerance percent defective (LTPD):" with value "1.", both followed by a percentage sign.
- Lot size:** A text box with value "10000".
- Process Sigma:** Two radio buttons: "Assume known value" (selected) and "Use sample estimate".
- Desired Features:** Two text boxes: "Producer's risk (alpha):" with value "5." and "Consumer's risk (beta):" with value "10.", both followed by a percentage sign.
- Current Plan:** Two text boxes: "Sample size (n):" and "Critical distance (k):", both empty.
- Buttons:** "OK", "Cancel", and "Help" at the bottom.

- **Action** - defines the type of action desired:
 1. *Create OC Plan* - creates a plan that controls the probability of accepting a lot at both the AQL and the LTPD. Rejected lots are sent back to the producer without being rectified.
 2. *Create AOQL Plan* - creates a plan that minimizes the total number of inspected units at a selected fraction of non-conforming items while insuring that the maximum percentage of non-conforming items accepted does not exceed a specified value. Rejected lots are subjected to 100% inspection and rectified.
 3. *Create LTPD Plan* - creates a plan that minimizes the total number of inspected units at a selected fraction of non-conforming items while controlling the probability of accepting a lot at the LTPD. Rejected lots are subjected to 100% inspection and rectified.
 4. *Analyze Existing Plan* - computes the operating characteristic curve for a sampling plan specified by the user.
- **Quality Levels** - defines the percentage defective in “good” and “bad” lots:

1. **AQL** (*acceptable quality level*) - the poorest level of quality that the consumer finds acceptable *on average*.
 2. **LTPD** (*lot tolerance percent defective*) - the poorest level of quality that the consumer is willing to tolerate in any given lot.
- **Lot Size** - the number of items N in the lot.
 - **Process Sigma** – whether to assume that the value of σ is known or will be replaced by the sample estimate s .
 - **Desired Features** - characteristics desired of the sampling plan, depending upon the type of plan selected:

<i>Type of Plan</i>	<i>Feature #1</i>	<i>Feature #2</i>
OC Plan	Producer's risk α - the probability of rejecting a lot with a percent defective equal to the AQL.	Consumer's risk β - the probability of accepting a lot with a percent defective equal to the LTPD.
AOQL Plan	Average percent defective at which inspection will be minimized.	The AOQL or maximum % of non-conforming items accepted after rectification
LTPD Plan	Average percent defective at which inspection will be minimized.	Consumer's risk β - the probability of accepting a lot with a percent defective equal to the LTPD.

- **Current Plan** - If *Analyze Existing Plan* is selected, the sample size n and critical distance k of the plan to be analyzed.

Analysis Summary

The *Analysis Summary* displays the generated plan:

Acceptance Sampling for Variables

Lot size: 10000

Desired features

Producer's risk (alpha): 5.0%

Consumer's risk (beta): 10.0%

Generated plan

Sample size (n) = 138

Critical distance (k) = 2.43545

Plan attributes

Acceptable quality level (AQL): 0.5%

Producer's risk (alpha) = 4.95538%

Lot tolerance percent defective (LTPD): 1.0%

Consumer's risk (beta) = 9.99992%

Average Outgoing Quality Limit (AOQL) = 0.48748% at 0.567986% defective

Average Total Inspection (ATI) =

626.7 units per lot at the AQL

1417.4 units per lot at the AOQL

9013.81 units per lot at the LTPD

There are several important sections of the output:

- **Desired Features** - summarizes the user-specified features upon which the plan is based. In the above example, the plan was constructed so as to have a producer's risk of no more than 5% and a consumer's risk of no more than 10%.
- **Generated plan** - shows the smallest sampling plan that has the desired features. In the example, $n = 138$ items are to be sampled from the lot of $N = 10,000$ and the lot accepted if the sample mean is at least 2.43545 standard deviations inside of all specification limits.
- **Plan Attributes** - exact results for the generated plan. This includes:

Producer's risk at the AQL - probability of rejecting a "good" lot.

Consumer's risk at the LTPD - probability of accepting a "bad" lot.

Average Outgoing Quality Limit - Assuming that rejected lots are 100% inspected and that any non-conforming items are replaced by good items, this is the maximum fraction of non-conforming items that are accepted.

Average Total Inspection - assuming that rejected lots are 100% inspected and that any non-conforming items are replaced by good items, this is the average percentage of items in a lot that will be inspected.

The plan generated in the output above would be implemented as follows:

1. A sample of $n = 138$ items will be taken from each lot of $N = 10,000$ items.

2. The sample mean $\bar{x}$ and sample standard deviation s will be computed.
3. Z scores will then calculated from:

$$Z_L = \frac{\bar{x} - LSL}{\sigma} \quad Z_U = \frac{USL - \bar{x}}{\sigma} \quad (1)$$

if the process standard deviation σ is known, or from

$$Z_L = \frac{\bar{x} - LSL}{s} \quad Z_U = \frac{USL - \bar{x}}{s} \quad (2)$$

if the process sigma is to be estimated from the data.

4. If both Z-scores are greater than k , the lot is accepted. Otherwise, it is either sent back to the producer or rectified by measuring every item and replacing any non-conforming items with conforming ones.

A typical example is a bottle manufacturer who produces lots of 10,000 bottles, each of which has a lower specification on breaking strength of 200 psi. From each lot of $N = 10,000$ bottles, a sample of $n = 138$ bottles would be taken and the strengths of each calculated. Assuming that σ is known, the average of the sample must be such that

$$Z_L = \frac{\bar{x} - LSL}{\sigma} \geq 2.435 \quad (3)$$

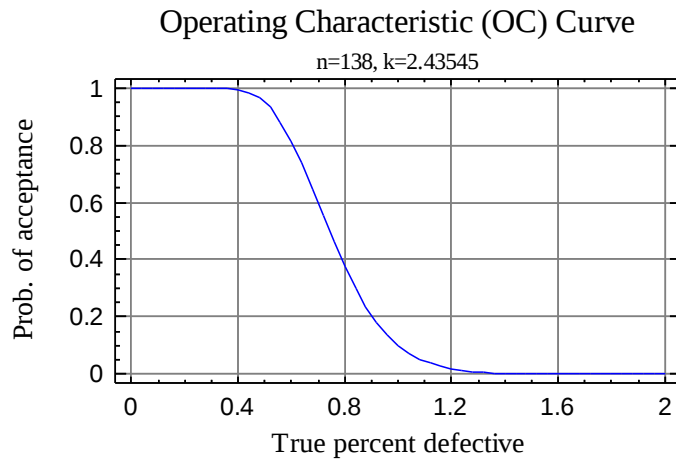
or

$$\bar{x} \geq 200 + 2.435\sigma \quad (4)$$

Note: If σ is estimated from the data rather than assumed to be known, the required sample size is a much larger $n = 546$, which may be verified by setting the *Process sigma* field to *Use sample estimate* on the data input dialog box.

OC Curve

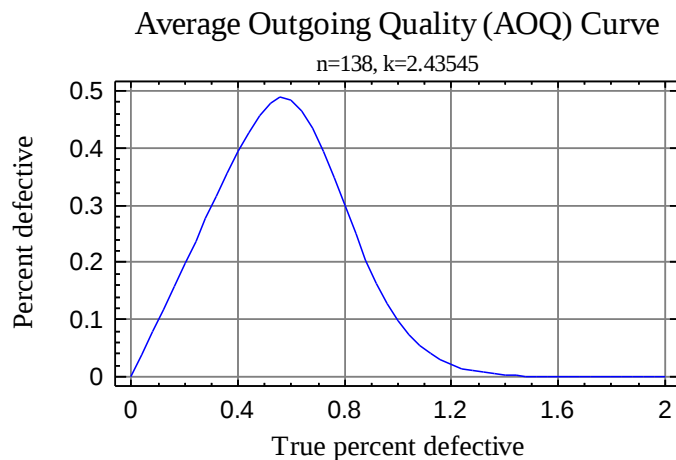
The *OC Curve* shows the probability that a lot with a percentage of non-conforming items equal to 1000% will be rejected by the current sampling plan:



For an *OC Plan* such as was generated for the current example, the OC Curve passes through $(1-\alpha)$ at the AQL and β at the LTPD.

AOQ Curve

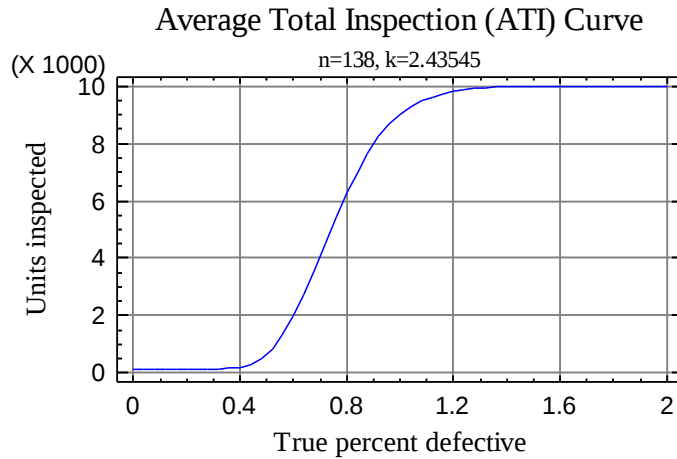
The AOQ curve shows the average outgoing quality of lots with a percentage of non-conforming items equal to 1000% when subjected to the current sampling plan:



The curve peaks at the AOQL.

ATI Curve

The ATI curve shows the average number of items inspected for lots with a percentage of non-conforming items equal to 1000% when subjected to the current sampling plan:



The ATI curve assumes that rejected lots are rectified.

Calculations

Acceptance Probability

The probability of accepting a lot containing a fraction non-conforming items equal to θ is computed from the standard normal distribution according to

$$P(\text{accept}|\theta) = \Phi(Z_\theta \sqrt{n}) \quad (5)$$

if σ is known and from

$$P(\text{accept}|\theta) = \Phi\left(\frac{Z_\theta}{\sqrt{\frac{1}{n} + \frac{k^2}{2n}}}\right) \quad (6)$$

if σ is estimated from the data, where Z_θ is the value of the standard normal distribution leaving an area of θ in the upper tail of the curve.

Producer's Risk

$$\alpha = P(\text{reject}|AQL) = 1 - P(\text{accept}|AQL) \quad (7)$$

Consumer's Risk

$$\beta = P(\text{accept} | LTPD) \quad (8)$$

Average Outgoing Quality

$$AOQ(\theta) = \theta P(\text{accept} | \theta) \left(\frac{N-n}{N} \right) \quad (9)$$

Average Outgoing Quality Limit

$$AOQL = \max_{\theta} \left[\theta P(\text{accept} | \theta) \left(\frac{N-n}{N} \right) \right] \quad (10)$$

Average Total Inspection

$$ATI(\theta) = n + (1 - P(\text{accept} | \theta))(N - n) \quad (11)$$

Reference

Montgomery, D. C. (2005). Introduction to Statistical Quality Control, 5th edition. New York: John Wiley and Sons.